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Thermodynamic Pareto optimization of turbojet engines using multi-objective genetic algorithms

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Abstract

Multi-objective genetic algorithms (GAs) are used for Pareto approach optimization of thermodynamic cycle of ideal turbojet engines. On this behalf, a new diversity preserving algorithm is proposed to enhance the performance of multi-objective evolutionary algorithms (MOEAs) in optimization problems with more than two objective functions. The important conflicting thermodynamic objectives that have been considered in this work are, namely, specific thrust (*ST*), thrust-specific fuel consumption (*TSFC*), propulsive efficiency (η_p), and thermal efficiency (η_t). In this paper, different pairs of these objective functions have been selected for two-objective optimization processes. Moreover, these objectives have also been considered for a four-objective optimization problem using the new diversity preserving algorithm of this work. The comparison results demonstrate the superiority of the new algorithm in preserving the diversity of non-dominated individuals and the quality of Pareto fronts in both two-objective and 4-objective optimization processes. Further, it is shown that some interesting and important relationships among optimal objective functions and decision variables involved in the thermodynamic cycle of turbojet engines can be discovered consequently. Such important relationships as useful optimal design principles would not have been obtained without the use of a multi-objective optimization approach. It is also demonstrated that the results of four-objective optimization can include those of two-objective optimization and, therefore, provide more choices for optimal design of thermodynamic cycle of ideal turbojet engines.

Keywords: Thermodynamics; Pareto optimization; Jet engines; Multi-objective optimization; Genetic algorithms

1. Introduction

Optimization in engineering design has always been of great importance and interest particularly in solving complex real-world design problems. Basically, the optimization process is defined as finding a set of values for a vector of design variables so that it leads to an optimum value of an objective or cost function. In such single-objective optimization problems, there may or may not exist some constraint functions on the design variables and they are respectively referred to as constrained or unconstrained optimization problems. There are many calculus-based methods including gradient approaches to search for mostly local optimum solutions and these are well documented in [1,2]. However, some basic difficulties in the gradient methods such as their strong dependence on the initial guess can cause them to find a local optimum rather than a global one. This has led to other heuristic optimization methods, particularly genetic algorithms (GAs) being used extensively during the last decade. Such nature-inspired evolutionary algorithms [3, 4] differ from other traditional calculus based techniques. The main difference is that GAs work with a population of candidate solutions, not a single point in search space. This helps significantly to avoid being trapped in local optima [5] as long as the diversity of the population is well preserved. In multi-objective optimization problems, there are several objective or cost functions (a vector of objectives) to be optimized (minimized or maximized) simulta-

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Nomenclature

M_0	flight Mach number	τ_r	total static temperature ratio at inlet
T_0	inlet temperature . . . . K	τ_t	burner exit/inlet total temperature ratio
a_0	velocity of sound at inlet . . . . $m \cdot s^{-1}$	τ_λ	burner exit total enthalpy/inlet total enthalpy
$\dot{m}_0$	mass flow rate . . . . $kg \cdot s^{-1}$	f	fuel/air ratio
R	gas constants . . . . $J \cdot kg^{-1} \cdot K^{-1}$	F	thrust . . . . N
g_c	Newton's constant . . . . $kg\text{-}m \cdot (N\text{-}s^2)^{-1}$	ST	specific thrust ($F/\dot{m}_0$)
V_0	air velocity at inlet . . . . $m \cdot s^{-1}$	$TSFC$	thrust-specific fuel consumption
V_9	gas velocity at exit . . . . $m \cdot s^{-1}$	η_P	propulsive efficiency
h_{PR}	heating value . . . . $kJ \cdot kg^{-1}$	η_T	thermal efficiency
γ	ratio of specific heats	X^*	vector of optimal design variables
C_P	specific heat at constant pressure . $kJ \cdot kg^{-1} \cdot K^{-1}$	$F(X)$	vector of objective functions
T_{t4}	burner exit total temperature . . . . K	$\mathcal{P}^*$	Pareto set (set of decision variables)
π_c	compressor pressure ratio	$\mathcal{PF}^*$	Pareto front (set of objective functions)
τ_c	compressor exit total temperature/compressor inlet total temperature		

neously. These objectives often conflict with each other so that as one objective function improves, another deteriorates. Therefore, there is no single optimal solution that is best with respect to all the objective functions. Instead, there is a set of optimal solutions, well-known as Pareto optimal solutions [6–9], which distinguishes significantly the inherent natures between single-objective and multi-objective optimization problems. V. Pareto (1848–1923) was the French–Italian economist who first developed the concept of multi-objective optimization in economics [10]. The concept of a Pareto front in the space of objective functions in multi-objective optimization problems (MOPs) stands for a set of solutions that are non-dominated to each other but are superior to the rest of solutions in the search space. Evidently, changing the vector of design variables in such a Pareto optimal solutions consisting of these non-dominated solutions would not lead to the improvement of all objectives simultaneously. Consequently, such change leads to a deterioration of at least one objective to an inferior one. Thus, each solution of the Pareto set includes at least one objective inferior to that of another solution in that Pareto set, although both are superior to others in the rest of search space. The inherent parallelism in evolutionary algorithms makes them suitably eligible for solving MOPs. The early use of evolutionary search is first reported in 1960s by Rosenberg [11]. Since then, there has been a growing interest in devising different evolutionary algorithms for MOPs. Among these methods, the vector evaluated genetic algorithm (VEGA) proposed by Schaffer [12], Fonseca and Fleming's genetic algorithm (FFGA) [7], non-dominated sorting genetic algorithm (NSGA) by Srinivas and Deb [6], and strength Pareto evolutionary algorithm (SPEA) by Zitzler and Thiele [13], and the Pareto archived evolution strategy (PAES) by Knowles and Corne [14] are the most important ones. A very good and comprehensive survey of these methods has been presented in [15–17]. Coello [18] has also presented an Internet based collection of many papers as a very good and easily accessible literature resource. Basically, both NSGA and FFGA as Pareto-based approaches use the revolutionary non-dominated sorting procedure originally proposed by Goldberg [3]. There are two important issues that have to be considered in such evolutionary multi-objective optimization methods: driving the search towards the true Pareto-optimal set or front and preventing premature convergence or maintaining the genetic diversity within the population [19]. The lack of elitism was also a motivation for modification of that algorithm to NSGA-II [20] in which a direct elitist mechanism, instead of a sharing mechanism, has been introduced to enhance the population diversity. This modified algorithm represents the state-of-the-art in evolutionary MOPs [21]. A comparison study among SPEA and other evolutionary algorithms on several problems and test functions showed that SPEA clearly outperforms the other multi-objective EAs [22]. Some further investigations reported in reference [19] demonstrated, however, that the elitist variant of NSGA (NSGA-II) equals the performance of SPEA. Despite its popularity and effectiveness, NSGA-II has some drawbacks that have been accordingly modified in this paper and will be presented in the following sections.

In thermal systems, like many other real-world engineering design problems, there are many complex optimisation design problems [23] which can also be multi-objective in nature. The objectives in thermal systems are usually conflicting and non-commensurable, and thus Pareto solutions provide more insights into the competing objectives. Recently, there has been a growing interest in evolutionary Pareto optimization in thermal systems. A thermoeconomic analysis has been performed by Toffolo and Lazzaretto [24] in which two exergic and economic issues in a cogeneration power plant have been considered as conflicting objectives. A similar point of view has also been considered by Wright et al. [25] in a multi-criterion optimization of a thermal design problem for a building. A monetary multi-objective optimization of a combined cycle power system has been studied by Roosen et al. [26]. An application of GA for thermodynamic optimization of turbofan engines has been performed by Homaifar et al. [27]. This study includes a single-objective optimization by GA for two objectives, namely, specific thrust and overall efficiency. They combined the results of the single-objective optimizations to find the interactions of the objective functions in a multi-objective sense.

In this paper, an optimal set of design variables in turbojet engines, namely, the input flight Mach number M_o, the pressure ratio of the compressor π_c, and the turbine inlet temperature T_{4t} are found using a Pareto approach to multi-objective optimization. In this study, different pairs of conflicting objectives in an ideal subsonic turbojet engines are selected for optimization. These include some combinations of thermal efficiency (η_t) and propulsive efficiency (η_p) together with thrust-specific fuel consumption (*TSFC*) and specific thrust (*ST*). In this way, a new diversity preserving algorithm called ε-elimination diversity algorithm is proposed to enhance the performance of NSGA-II in terms of diversity of population and Pareto fronts. The modified algorithm can be used for multi-objective optimization with more than two objectives. Consequently, four-objective optimization of turbojet engines is conducted considering η_t, η_p, *TSFC*, and *ST* as competing objectives. The superiority of the ε-elimination diversity preserving mechanism is shown, compared to that of NSGA-II.

2. Multi-objective optimization

Multi-objective optimization, which is also called multicriteria optimization or vector optimization, has been defined as finding a vector of decision variables satisfying constraints to give acceptable values to all objective functions [8,28]. In general, it can be mathematically defined as: find the vector $X^* = [x_1^*, x_2^*, \ldots, x_n^*]^{\mathrm{T}}$ to optimize

$$F(X) = \left[f_1(X), f_2(X), \ldots, f_k(X)\right]^{\mathrm{T}} \tag{1}$$

subject to m inequality constraints

$$g_i(X) \leqslant 0, \quad i = 1, \ldots, m \tag{2}$$

and p equality constraints

$$h_j(X) = 0, \quad j = 1, \ldots, p \tag{3}$$

where $X^* \in \Re^n$ is the vector of decision or design variables, and $F(X) \in \Re^k$ is the vector of objective functions, which must each be either minimized or maximized. However, without loss of generality, it is assumed that all objective functions are to be minimized. Such multi-objective minimization based on Pareto approach can be conducted using some definitions:

2.1. Definition of Pareto dominance

A vector $U = [u_1, u_2, \ldots, u_k] \in \Re^k$ is dominant to vector $V = [v_1, v_2, \ldots, v_k] \in \Re^k$ (denoted by $U \prec V$) if and only if $\forall i \in \{1, 2, \ldots, k\}$, $u_i \leqslant v_i \wedge \exists j \in \{1, 2, \ldots, k\}$: $u_j < v_j$. In other words, there is at least one u_j which is smaller than v_j whilst the remaining u's are either smaller or equal to corresponding v's.

2.2. Definition of Pareto optimality

A point $X^* \in \Omega$ (Ω is a feasible region in $\Re^n$ satisfying Eqs. (2) and (3)) is said to be Pareto optimal (minimal) with respect to all $X \in \Omega$ if and only if $F(X^*) \prec F(X)$. Alternatively, it can be readily restated as $\forall i \in \{1, 2, \ldots, k\}$, $\forall X \in \Omega - \{X^*\} f_i(X^*) \leqslant f_i(X) \wedge \exists j \in \{1, 2, \ldots, k\}$: $f_j(X^*) < f_j(X)$. In other words, the solution X^* is said to be Pareto optimal (minimal) if no other solution can be found to dominate X^* using the definition of Pareto dominance.

2.3. Definition of a Pareto set

For a given MOP, a Pareto set $\mathcal{P}^*$ is a set in the decision variable space consisting of all the Pareto optimal vectors $\mathcal{P}^* = \{X \in \Omega \mid \nexists X' \in \Omega\colon F(X') \prec F(X)\}$. In other words, there is no other X' as a vector of decision variables in Ω that dominates any $X \in \mathcal{P}^*$.

2.4. Definition of a Pareto front

For a given MOP, the Pareto front $\mathcal{PF}^*$ is a set of vector of objective functions which are obtained using the vectors of decision variables in the Pareto set $\mathcal{P}^*$, that is $\mathcal{PF}^* = \{F(X) = (f_1(X), f_2(X), \ldots, f_k(X))\colon X \in \mathcal{P}^*\}$. In other words, the Pareto front $\mathcal{PF}^*$ is a set of the vectors of objective functions mapped from $\mathcal{P}^*$.

Evolutionary algorithms have been widely used for multi-objective optimization because of their natural properties suited for these types of problems. This is mostly because of their parallel or population-based search approach. Therefore, most of the difficulties and deficiencies within the classical methods in solving multi-objective optimization problems are eliminated. For example, there is no need for either several runs to find all individuals of the Pareto front or quantification of the importance of each objective using numerical weights. In this way, the original non-dominated sorting procedure given by Goldberg [3] was the catalyst for several different versions of multi-objective optimization algorithms [6,7]. However, it is very important that the genetic diversity within the population be preserved sufficiently. This main issue in MOPs has been addressed by many related research works [19]. Consequently, the premature convergence of MOEAs is prevented and the solutions are directed and distributed along the true Pareto front if such genetic diversity is well provided. The Pareto-based approach of NSGA-II [20] has been used recently in a wide area of engineering MOPs because of its simple yet efficient non-dominance ranking procedure in yielding different level of Pareto frontiers. However, the crowding approach in such state-of-the-art MOEA [21] is not efficient as a diversity-preserving operator, particularly in problems with more than two objective functions. In order to show this issue more clearly, some basics of NSGA-II are now represented. Fig. 1 illustrates the main procedure of selecting individuals from the entire population R_t to construct the next parent population R_{t+1}. The entire population R_t is simply the current parent population P_t plus its offspring population Q_t which is created from the parent population P_t by using usual genetic operators. The selection is based on non-dominated sorting procedure which is used to classify the entire population R_t according to increasing order of dominance [20]. Thereafter, the best Pareto fronts from the top of the sorted list is transferred to create the new parent population P_{t+1} which is half the size of the entire population R_t. Therefore, it should be noted that all the individuals of a certain front cannot be accommodated in the new parent population because of space, as shown in Fig. 1. In order to choose an exact number of individuals of that particular front, a crowded comparison operator is used in NSGA-II to find the best solutions to fill the rest of the new parent population slots. The crowded comparison procedure is based on density estimation of solutions surrounding a particular solution in a population or front. In this way, the solutions of a Pareto front are first sorted in each objective direction in the ascending order of that objective value. The crowding distance is then assigned equal to the half of the perimeter of the enclosing hyper-box. The sorting procedure is then repeated for other objectives and the overall crowding distance is calculated as the sum of the crowding distances from all objectives. The less crowded non-dominated individuals of that particular Pareto front are then selected to fill the new parent population. It must be noted that, in a *two-objective* Pareto optimization, if the solutions of a Pareto front are sorted in a *decreasing* order of importance to one objective, these solutions are then automatically ordered in an *increasing* order of importance to the second objective. In other words, the hyper-boxes surrounding an individual solution remain unchanged in the objective-wise sorting procedure of the crowding distance of NSGA-II in the two-objective Pareto optimization problem. However, in multi-objective Pareto optimization problem with more than two objectives, such sorting procedure of individuals based on each objective in this algorithm will cause different enclosing hyper-boxes. Thus, the overall crowding distance of an individual computed in this way may not exactly reflect the true measure of diversity or crowding property for the multi-objective Pareto optimization problems with more than two objectives.

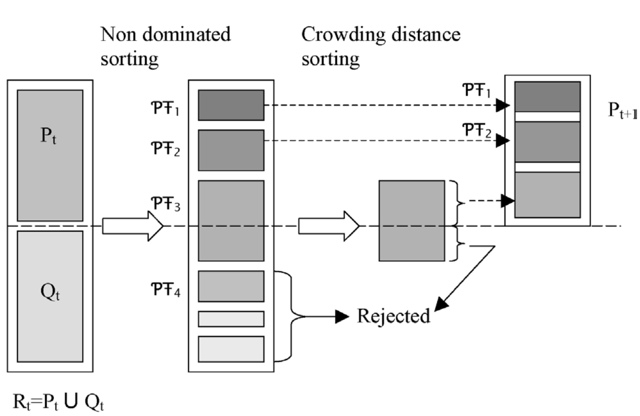

Fig. 1. Basics of NSGA-II procedure.

In this work, a new method is presented which modifies NSGA-II so that it can be safely used for any number of objective functions (particularly for more than two objectives) in MOPs. The modified MOEA is then used for a four-objective thermodynamic optimization of subsonic turbojet engines and the results are compared with those of the original NSGA-II.

3. The ε-elimination diversity algorithm

In the ε-elimination diversity approach that is used to replace the crowding distance assignment approach in NSGA-II [20], all the clones and/or ε-similar individuals based on Euclidean norm of two vectors are recognized and simply eliminated from the current population. Therefore, based on a pre-defined value of ε as the elimination threshold ($\varepsilon = 0.001$ has been used in this paper) all the individuals in a front within this limit of a particular individual are eliminated. It should be noted that such ε-similarity must exist both in the space of objectives and in the space of the associated design variables. This will ensure that very different individuals in the space of design variables having ε-similarity in the space of objectives will not be eliminated from the population. The pseudo-code of the ε-elimination approach is depicted in Fig. 2. Evidently, the clones or ε-similar individuals are replaced from the population with the same number of new randomly generated individuals. Mean-

Pseudo-code of e-elimination
ε-elim=ε-elimination (*pop*) //*pop* includes design variables and objective functions.
define ε //Define elimination threshold.
get k ($k = 1$ for the first front) //Front No.
$i = l$
until $i + 1 < pop\_size$
  $j = i + 1$
  until $j < pop\_size$
  IF $\{\|F(X(i), F(X(j))\| < \varepsilon \wedge \|X(i), X(j)\| < \varepsilon\}$
    $F(X(i)), F(X(j)) \in \mathcal{PF}_k^* \quad X(i), X(j) \in \mathcal{P}_k^*$
  THEN $pop = pop \backslash pop(j)$ //Remove the ε-similar individual.
r_new_ind = make_new_random_individual
 //Generate new random individual.
pop = pop $\cup$ r_new_ind //Add the newly generated individual.
  end
end

Fig. 2. Pseudo-code of ε-elimination for preserving genetic diversity.

while, this will additionally help to explore the search space of the given MOP more efficiently.

4. Multi-objective thermodynamic optimization of turbojet engines with two design variables

4.1. Thermodynamic model of turbojet engines

Turbojet engines use air as the working fluid and produce thrust based on the variation of kinetic energy of burnt gases after combustion. The study of the thermodynamic cycle of a turbojet engine involves different thermo-mechanical aspects such as specific thrust, thermal and propulsive efficiencies, and thrust-specific fuel consumption. A detailed description of the thermodynamic analysis and equations [29] of ideal turbojet engines is given in Appendix A. This elementary thermodynamic model is sufficient to capture the principles of behaviour and interactions among different input and output parameters in a multi-objective optimal sense. Furthermore, this provides a suitable real-world engineering benchmark for comparing purpose between MOEA using the new diversity preserving mechanism of this work with NSGA-II.

The input parameters involved in such thermodynamic analysis in an ideal turbojet engine given in Appendix A are flight Mach number (M_0), input air temperature (T_0, K), specific heat ratio (γ), heating value of fuel (h_{PR}, $\mathrm{kJ \cdot kg^{-1}}$), exit burner total temperature (T_{t4}, K), and compressor pressure ratio, π_c. The output parameters involved in the thermodynamic analysis in the ideal turbojet engine given in Appendix A are, specific thrust, (ST, $\mathrm{N \cdot kg^{-1} \cdot s^{-1}}$), fuel-to-air ratio ($f$), thrust-specific fuel consumption ($TSFC$, $\mathrm{kg \cdot s^{-1} \cdot N^{-1}}$), thermal efficiency ($\eta_t$), and propulsive efficiency (η_p). However, in the multi-objective optimization study, some input parameters are already known or assumed as, $T_0 = 216.6$ K, $\gamma = 1.4$, $h_{\mathrm{pr}} = 48000\ \mathrm{kJ \cdot kg^{-1}}$, and $T_{t4} = 1666$ K. The input flight Mach number $0 < M_0 \leqslant 1$ and the compressor pressure ratio $1 \leqslant \pi_c \leqslant 40$ are considered as design variables to be optimally found based on multi-objective optimization of 4 output parameters, namely, ST, $TSFC$, η_t, and η_p.

4.2. Two-objective thermodynamic optimization of turbojet engines

In order to investigate the optimal thermodynamic behaviour of subsonic turbojet engines, 5 different sets, each including two objectives of the output parameters, are considered individually. Such pairs of objectives to be optimized separately have been chosen as $(\eta_p, TSFC)$, (η_p, ST), $(\eta_t, TSFC)$, (η_t, ST), and (η_p, η_t). Evidently, it can be observed that η_p, η_t, and ST are maximized whilst $TSFC$ is minimized in those sets of objective functions. A population size of 100 has been chosen in all runs with crossover probability P_c and mutation probability P_m as 0.8 and 0.02, respectively.

The results of the two-objective optimizations considering those 5 different combinations of objectives are summarized in Table 1. Some Pareto fronts of each pair of two objectives have been shown through Figs. 3 and 4 using both the approach of this work and that of NSGA-II. It is clear from these figures that choosing appropriate values for the decision variables, namely flight Mach number (M_0) and pressure ratio (π_c), to obtain a better value of one objective would normally cause a worse value of another objective. However, if the set of decision variables is selected based on each of a Pareto front, it will lead to the best possible combination of that pair of objectives. In other words, if any other pair of decision variables M_0 and π_c is chosen, the corresponding values of the particular pair of objectives will locate a point inferior to that Pareto front. The inferior area in the space of objective functions (plane in these cases) for Figs. 3 and 4 are in fact bottom/left sides. A better diversity of results obtained using the approach of this work than those of NSGA-II can also observed in these figures. Evidently, Figs. 3 and 4 reveal some important and interesting optimal relationships among the thermodynamic parameters in the ideal thermodynamic cycle of turbojet engines that may not have been found without a multi-objective optimization approach. Such relationships have been called a worthwhile task for a designer by Deb in [30]. These figures and the associated values of the decision variables and the objective functions given in Table 1 simply cover all the 4 objectives studied in the two-objective Pareto optimization. However, other pairs of objective functions in the two-objective Pareto optimization together with their associated values of the de-

Table 1
Values of decision variables and objective functions in various two-objective optimizations

	Pairs of objectives in two-objective optimizations						
	$(\eta_p, TSFC)$		$(\eta_t, TSFC)$	(η_p, ST)		(η_t, ST)	(η_p, η_t)
	$0 < \eta_p \leqslant 0.39$ $2.1 \leqslant TSFC \times 10^5 \leqslant 2.43$	$0.4 < \eta_p \leqslant 0.55$ $3.16 \leqslant TSFC \times 10^5 \leqslant 6.8$	$0.65 \leqslant \eta_t \leqslant 0.7$ $2.1 \leqslant TSFC \times 10^5 \leqslant 2.43$	$0.41 < \eta_p \leqslant 0.5$ $515 \leqslant ST \leqslant 817$	$0 < \eta_p < 0.39$ $906 \leqslant ST \leqslant 1169$	$0.64 \leqslant \eta_t \leqslant 0.7$ $890 \leqslant ST \leqslant 1169$	$0.4 \leqslant \eta_p \leqslant 0.56$ $0.16 \leqslant \eta_t \leqslant 0.55$
Flight Mach number (M_0)	$0 < M_0 \leqslant 1$	1	$0 < M_0 \leqslant 1$	$0.85 \leqslant M_0 \leqslant 1$	$0 < M_0 \leqslant 1$	$0 < M_0 \leqslant 1$	1
Pressure ratio (π_c)	$\pi_c = 40$	$1.0 \leqslant \pi_c \leqslant 8.25$	$\pi_c = 40$	$1.2 \leqslant \pi_c \leqslant 4.28$	$13.5 \leqslant \pi_c \leqslant 39.3$	$37.3 \leqslant \pi_c \leqslant 40$	$1 \leqslant \pi_c \leqslant 8.78$

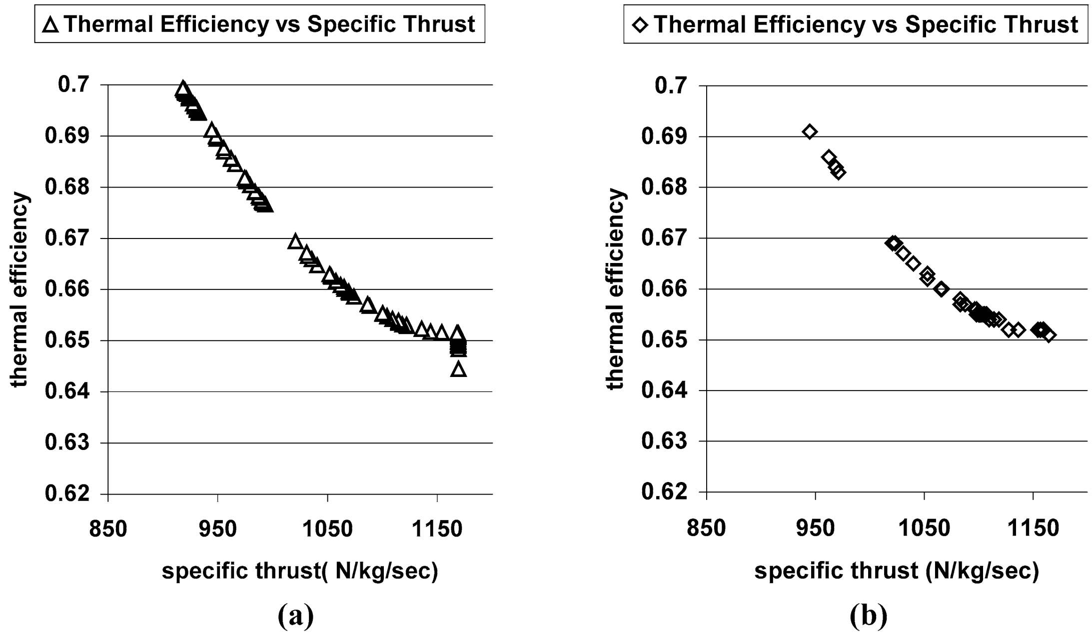

Fig. 3. Pareto front of thermal efficiency and specific thrust in 2-objective optimization: (a) ε-elimination approach; (b) NSGA-II.

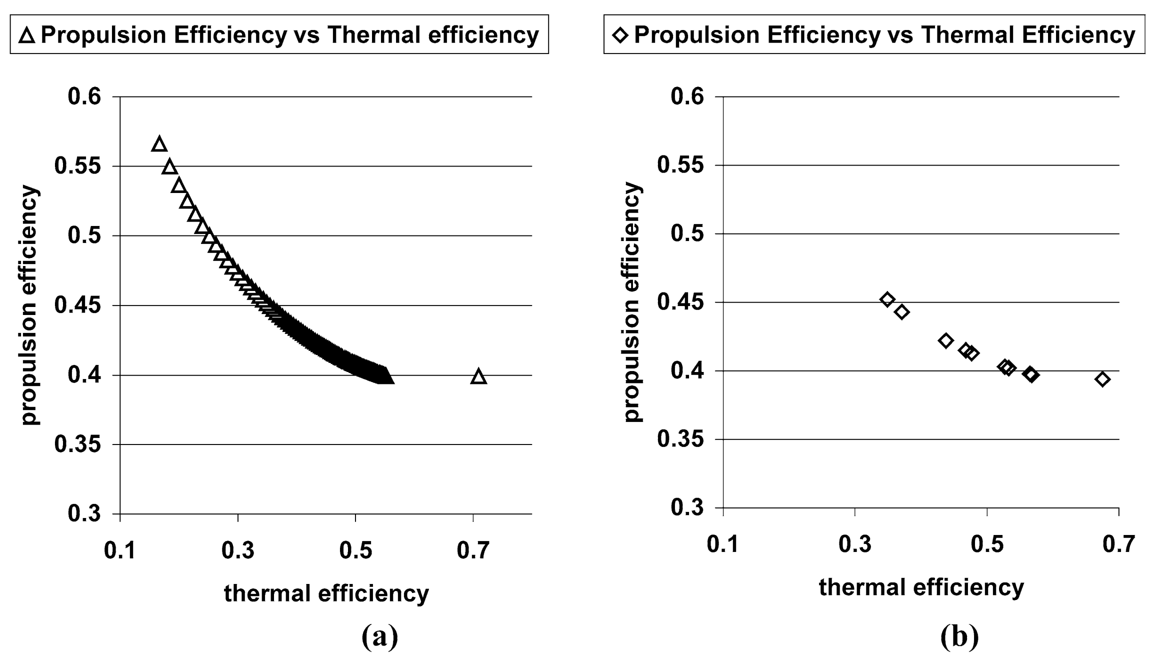

Fig. 4. Pareto front of propulsive efficiency and thermal efficiency in 2-objective optimization: (a) ε-elimination approach; (b) NSGA-II.

cision variables have been shown in Table 1. A careful investigation of these Pareto optimization results reveals some interesting and informative design aspects. It can be observed that a small value of pressure ratio ($\pi_c < 8.7$) is required in large value of Mach number ($0.85 < M_0 < 1$) when high value of η_p is important to the designer ($0.4 < \eta_p < 0.55$). In this case both *ST* and *TSFC* get their worse values (*ST* becomes smaller and *TSFC* becomes larger), whilst η_t varies between small and medium values ($0.16 < \eta_t < 0.55$) depending on the value of flight Mach number. However, with high value of pressure ratio ($37 < \pi_c < 40$) in a wide range of flight Mach number ($0 < M_0 < 1$), *TSFC*, *ST*, and η_t improve whilst η_p cannot be better than 0.4. The specific values of these objectives depend on the exact value of flight Mach number which have been given in Table 1. However, such important and worthwhile information can be simply discovered using a four-objective Pareto optimization, which will be presented in the next section.

Moreover, Figs. 3 and 4 also reveal some important and interesting optimal relationships of such objective functions in ideal thermodynamic cycle of turbojet engines that may have not been known without a multi-objective optimization

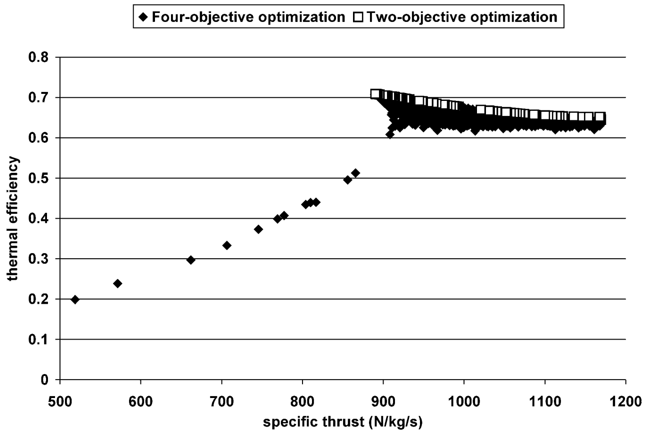

Fig. 5. Thermal efficiency variation with specific thrust in both 4-objective & 2-objective optimisation.

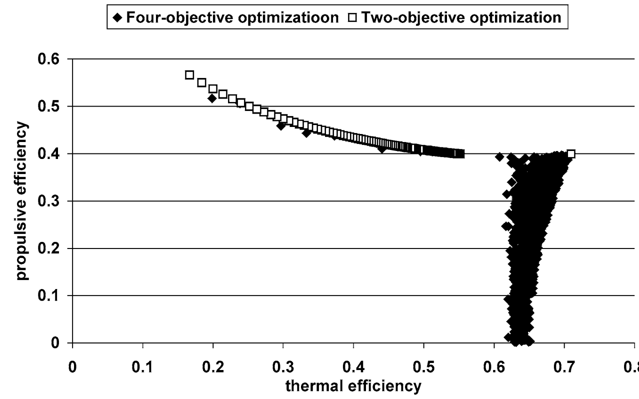

Fig. 6. Propulsive efficiency variation with thermal efficiency in both 4-objective & 2-objective optimization.

approach. For example, Fig. 3 demonstrates that the optimal behaviours of η_t with respect to ST can be readily represented by

$$\eta_t \propto (ST)^2 \tag{4}$$

Fig. 4 represents a non-linear optimal relationship of η_t and η_p in the form of

$$\eta_t \propto (\eta_p)^2 \tag{4}$$

It should be noted that these relationships, which have been obtained from the two-objective Pareto optimization results, are valid when the corresponding two-objective optimization of such functions is of importance to the designer and, in fact, demonstrates the optimal compromise of such pairs of objectives.

4.3. Four-objective thermodynamic optimization of turbojet engines

A multi-objective thermodynamic optimization including all four objectives simultaneously can offer more choices for a designer. Moreover, such 4-objective optimization can subsume all the 2-objective optimization results presented in the previous section. Therefore, in this section, four objectives, namely, *TSFC*, ST, η_p, and η_t, are chosen for multi-objective optimization in which ST, η_p, and η_t are maximized whilst *TSFC* is minimized simultaneously. A population size of 200 has been chosen with crossover probability P_c and mutation probability P_m as 0.8 and 0.02, respectively.

Fig. 5 depicts the non-dominated individuals in both 4-objective and previously obtained 2-objective optimization in the plane of $(\eta_t - ST)$. Such non-dominated individuals in both 4 and 2-objective optimization have alternatively been shown in the plane of $(\eta_p - \eta_t)$ in Fig. 6. It should be noted that there is a single set of individuals as a result of 4-objective optimization of *TSFC*, ST, η_p, and η_t that are shown in different planes together with the corresponding 2-objective optimization results. Therefore, there are some points in each plane that may dominate others in the same plane in the case of 4-objective optimization. However, these individuals are all non-dominated when considering all four objectives simultaneously. By careful investigation of the results of 4-objective optimization in each plane, the Pareto fronts of the corresponding two-objective optimization can now be observed in these figures. It can be readily observed that the results of such 4-objective optimization include the Pareto fronts of each 2-objective optimization and provide, therefore, more optimal choices for the designer.

The non-dominated individuals obtained in 4-objective optimization demonstrate some interesting behaviours in terms of design variables. Two different parts can be easily observed in Figs. 5 and 6. One of these parts which is less populated corresponds to high value of pressure ratio ($0.4 < \eta_p < 0.55$), unlike the rest of objective functions which all together degrades in their values simultaneously, that is, $3 < TSFC \times 10^5 < 6.3$, $515 < ST < 890$, $0.2 < \eta_t < 0.52$. The corresponding values of objectives for the second part can be given as, $0 < \eta_p < 0.4$, $2 < TSFC \times 10^5 < 3$, $900 < ST < 1169$, $0.6 < \eta_t < 0.71$ which can be appropriately chosen by the designer. Such facts would be very important to the designer to switch from one optimal solution to another for achieving different trade-off requirements of the objectives [30].

Additionally, there are some more profound optimal design relationships among the objective functions and the decision variables which have been discovered by the four-objective thermodynamic Pareto optimization of ideal turbojet engines. Such important optimal design facts could not have been found without the multi-objective Pareto optimization. Firstly, Fig. 7 shows the variation of 4 optimized objective functions ST, *TSFC*, η_p, and η_t with the pressure ratio. It can be seen that for pressure ratio less than 14, three objectives ST, *TSFC*, and η_t become worse, unlike η_p which gradually starts getting better. The slope of such degradation for ST, *TSFC*, and η_t becomes faster especially in *TSFC* and η_t when the pressure ratio becomes smaller than 6. However, for high pressure ratios, the variation of optimal values of *TSFC* and η_t are small whilst there are a wide range of selections for $\eta_p \approx 0.4$. Secondly, Fig. 8 demonstrates the

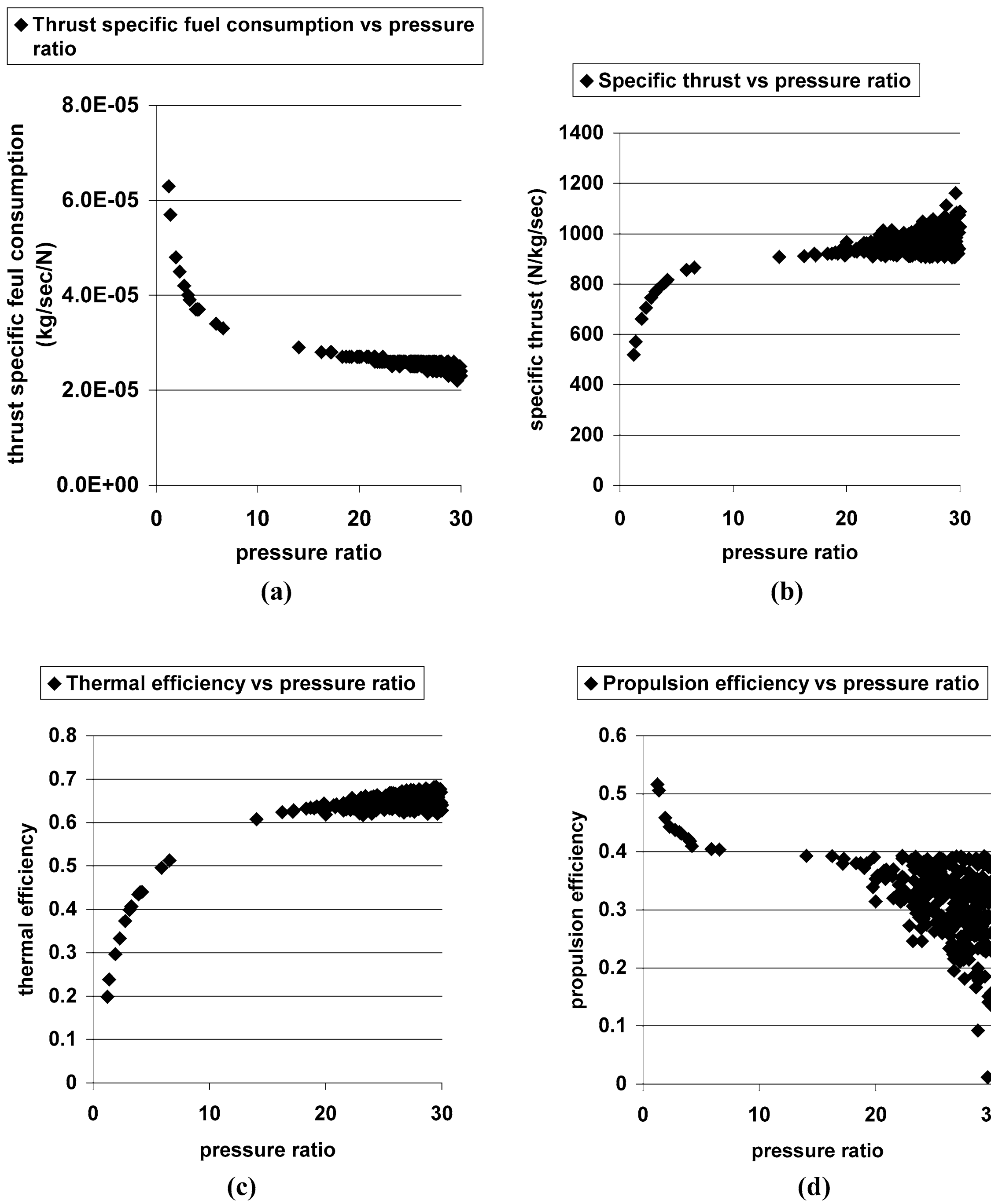

Fig. 7. Variation of four objective functions with compression ratio in 4-objective optimization.

behaviours of ST and η_p with respect to flight Mach number in high pressure ratios. It can be readily seen that the optimal values of ST changes linearly with M_0, that is

$$ST = -264.75M_0 + 1164.5 \tag{5}$$

with a R-squared value of 0.999. The optimal relationship of η_p with M_0 is non-linear and is represented as

$$\eta_p = -0.0977(M_0)^2 + 0.491M_0 + 0.0013 \tag{6}$$

with a R-squared value of 0.998.

Therefore, such multi-objective optimization of ST, $TSFC$, η_p, and η_t provide optimal choices of design variables based on Pareto non-dominated points.

Fig. 9 shows the diversity distribution of the non-dominated points using both the approach of this work and that of NSGA-II in the case of four-objective optimization. Such good diversity distribution is essential to reveal the true optimal relationship and Pareto frontiers of objectives and/or variables. A careful investigation of 4 objectives' values corresponding to the single point close to {0.5, 15} obtained by

NSGA-II reveals that it has been simply replaced by several dominant individuals obtained by the approach of this work.

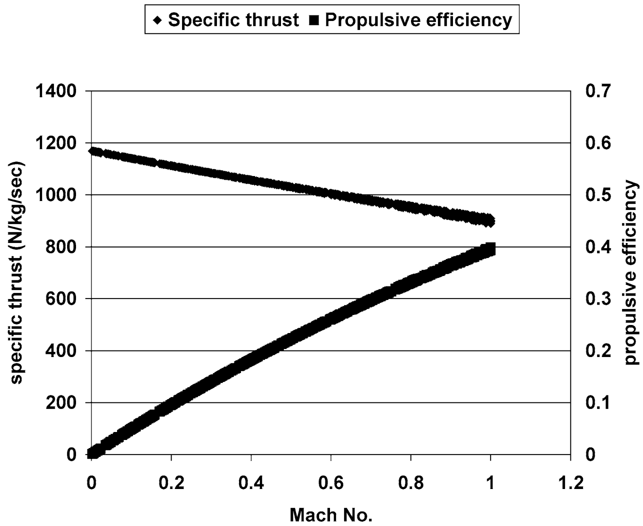

Fig. 8. Relationships of specific thrust & propulsive efficiency with flight Mach No. in 4-objective optimization.

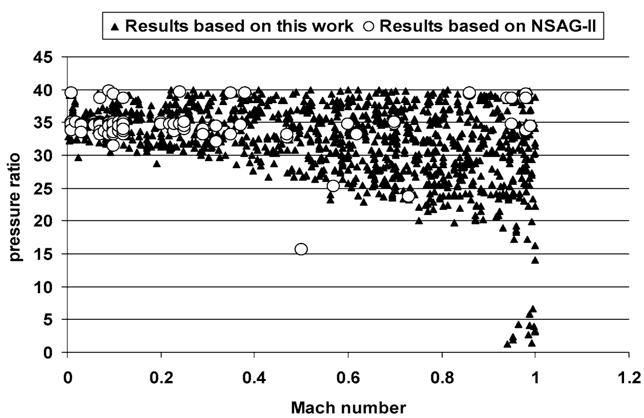

Fig. 9. Diversity comparison of design variables between this work and NSGA-II in 4-objective optimization of a turbojet engine.

5. Multi-objective thermodynamic optimization of turbojet engines with three design variables

The same procedure of the Pareto optimization both in two-objective and four-objective applied in Section 4 has been repeated in this section considering three design variables. The turbine inlet temperature T_{t4} which has been fixed to its maximum value of 1666 K in the previous section is now considered as a design variables in addition to the other two design variables, pressure ratio and flight Mach number. The lower limit of the turbine inlet temperature T_{t4} has been assumed 1400 K. Therefore, the input flight Mach number $0 < M_0 \leqslant 1$, the compressor pressure ratio $1 \leqslant \pi_c \leqslant 40$, and the turbine inlet temperature 1400 K $< T_{t4} <$ 1666 K are now considered as 3 design variables to be optimally found based on multi-objective optimization of 4 output parameters, namely, *ST*, *TSFC*, η_t, and η_p. Fig. 10(a) and (b) show the comparison of the results of two-objective optimization problems both in 2 design and 3 design variables. The results clearly show that the optimization procedure finds the best turbine inlet temperature T_{t4} as well as those for the other two decision variables according to the selected pair of the objectives. In Fig. 10(a) the best obtained T_{t4} corresponds to the lower limit of the turbine inlet temperature as the *TSFC* is one of the objective functions. In comparison with 2-design variables optimization, both propulsive and thermal efficiency (not shown) get better values in the same value of *TSFC* when T_{t4} is in its minimum value instead of the fixed maximum value of 1666 K in the case of 2 design variables. Similarly, the value of *TSFC* in the case of 3 design variables is less than that in the case of 2 design variables at the same values of both propulsive and ther-

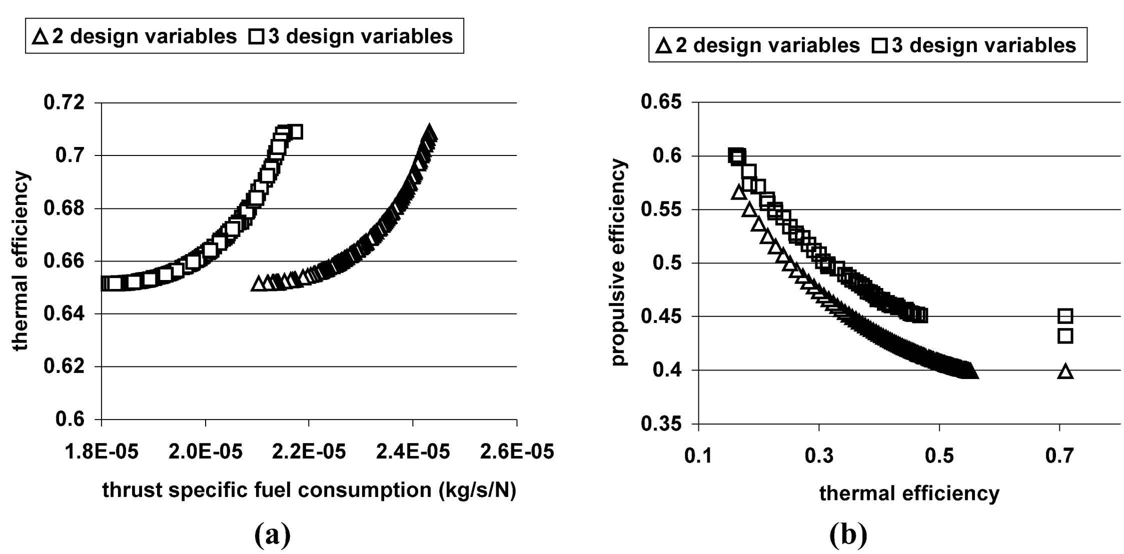

Fig. 10. Comparison graphs of two-objective optimization problems with 2 & 3 design variables: (a) propulsive efficiency vs. thermal efficiency; (b) thermal efficiency vs. *TSFC*.

mal efficiencies. Fig. 10(b) shows the improvement of the Pareto optimization of propulsive and thermal efficiencies in the case of 3 design variables when T_{t4} finds the minimum value as its best. It can be easily seen from this figure that the value of each efficiency in the case of 3 design variables is better than that in the case of 2 design variables.

These three design variables have also been considered in the case of four-objective Pareto optimization problem. In this case, the Pareto optimal solutions correspond to different values of T_{t4} including both its lower and upper limit. The Pareto fronts obtained in the case of 3 design variables have basically the same shapes of those in the case of 2 design variables but with a larger area. In fact, the Pareto fronts which have been found in the case of 2 design variables with the fixed value of T_{t4} of 1666 K are normally the boundary (either as the best or as the worst) of the solutions with 3 design variables. Figs. 11 and 12 show this situation in the case of 3 design variables in the comparison with the corresponding Fig. 8 of the case of 2 design variables. The same linear and non-linear relationships can be readily verified from these figures. The upper limit of the linear Pareto optimal relationship of *ST* versus the flight Mach number of the Fig. 11 corresponds to the one in Fig. 8. It can be seen that the lower limit of that figure has the same linear relationship which corresponds to the minimum T_{t4}. However, the lower limit of the non-linear relationship of the propulsive efficiency versus the flight Mach number in Fig. 12 corresponds to the maximum T_{t4} which is the same as in Fig. 8. This means that the propulsive efficiency in four-objective optimization can reach higher values with 3 design variables than that with 2 design variables when T_{t4} is fixed in its minimum value.

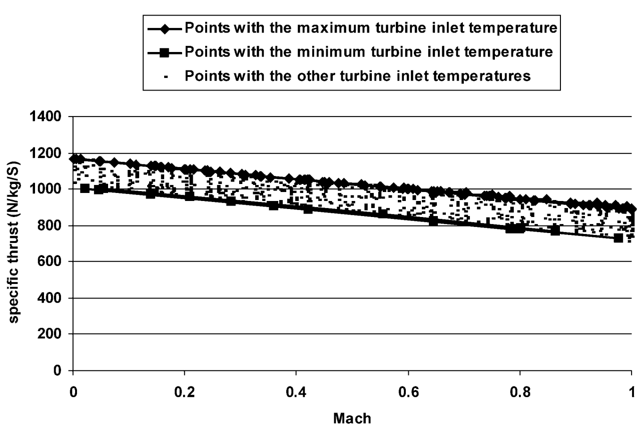

Fig. 11. Graph of specific thrust with flight Mach No. in 4-objective optimization with 3 design variables (pressure ratio, flight Mach number, turbine inlet temperature).

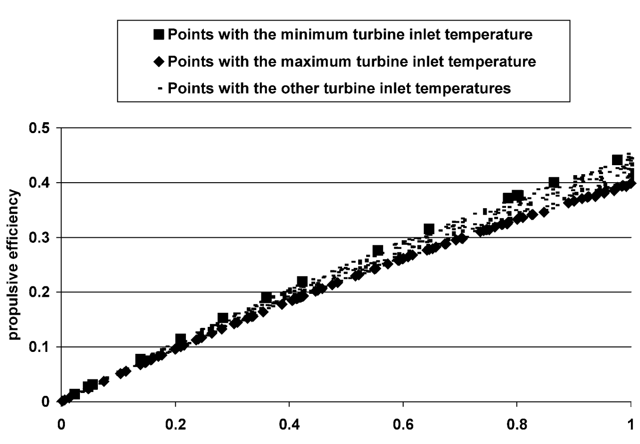

Fig. 12. Graph of propulsive efficiency with flight Mach No. in 4-objective optimization with 3 design variables (pressure ratio, flight Mach number, turbine inlet temperature).

6. Conclusion

A new diversity preserving mechanism called the ε-elimination algorithm has been proposed and successfully used with the Pareto approach of MOEAs for thermodynamic cycle optimization of ideal turbojet engines. It has been shown that the performance of this algorithm is superior to that of NSGA-II in terms of diversity and the quality of Pareto front obtained for both 2-objective and 4-objective optimization processes. Such multi-objective optimization led to the discovering of important relationships and useful optimal design principles in thermodynamic optimization of ideal turbojet engines both in the space of objective functions and decision variables. The evolutionary multi-objective optimization process has helped to discover important relationships with relatively few efforts of modeling preparation that would otherwise have required at least a very through mathematical analysis. If the underlying objective modeling becomes more complex (like deviating from the ideality of components behaviour) evolutionary multi-objective optimization process may even be expected to become the sole present-time means of attaining respective solutions. Further, it has been shown that the results of 4-objective optimization include those of 2-objective optimization in terms of Pareto frontiers and provide, consequently, more choices for optimal design.

Appendix A. Thermodynamic model of ideal turbojet engine

Assumptions: Inlet diffuser, compressor, turbine and exit nozzle, all operate isentropically.

No pressure loss in the burner. $f=(\text{fuel/air})\ll 1$, P_e(turbojet exit pressure) $=P_0$(ambient pressure), $C_P=1.004$ (kJ·kg^{-1}·K^{-1}) $T_0=216.6$ K, $\gamma=1.4$, $h_{\text{PR}}=48000$ kJ·kg^{-1}, $T_{t4}=1666$ K (in 2 design variables), $g_c=1$ (kg-m·(N-s^2)$^{-1}$).

Input parameters: M_0, T_0 (K), γ, c_P (kJ·kg^{-1}·K^{-1}), h_{PR} (kJ·kg^{-1}), T_{t4} (K), π_c.

Output parameters: $ST=\frac{F}{\dot m_0}$ (N·kg^{-1}·s^{-1}), *TSFC* (kg·s^{-1}·N^{-1}), η_t, η_P.

Equations:

$$R = \frac{\gamma - 1}{\gamma} c_p \tag{A.1}$$

$$a_0 = \sqrt{\gamma R g_c T_0} \tag{A.2}$$

$$\tau_r = 1 + \frac{\gamma - 1}{2} M_0^2 \tag{A.3}$$

$$\tau_\lambda = \frac{T_{t4}}{T_0} \tag{A.4}$$

$$\tau_c = (\pi_c)^{(\gamma-1)/\gamma} \tag{A.5}$$

$$\tau_t = 1 - \frac{\tau_r}{\tau_\lambda}(\tau_c - 1) \tag{A.6}$$

$$\frac{V_9}{a_0} = \sqrt{\frac{2}{\gamma - 1} \frac{\tau_\lambda}{\tau_r \tau_c} (\tau_r \tau_c \tau_t - 1)} \tag{A.7}$$

$$ST = \frac{F}{\dot{m}_0} = \frac{a_0}{g_c}\left(\frac{V_9}{a_0} - M_0\right) \tag{A.8}$$

$$f = \frac{c_P T_0}{h_{\mathrm{PR}}}(\tau_\lambda - \tau_r \tau_c) \tag{A.9}$$

$$SFC = \frac{f}{ST} \tag{A.10}$$

$$\eta_t = 1 - \frac{1}{\tau_r \tau_c} \tag{A.11}$$

$$\eta_p = \frac{2M_0}{V_9/a_0 + M_0} \tag{A.12}$$

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